[**Quasiperiodicity, periodicity, and chaos in injected Gunn oscillators.**]

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Abstract.

We observe a continuous spectrum in the video output of a diode detector which is driven by the sum of the output of an injected Gunn source and its injection signal. This may be evidence for chaos in this periodically forced non-linear system.

Both quasiperiodic motions and true chaotic motions of a nonlinear dynamical system result in motion which never repeats in the time domain. Thus the trajectory in phase space never closes on itself. The distinguishing feature of chaos is that the frequency spectrum, whilst not necessarily smooth, exists at all frequencies (or nearly all frequencies) over at least some range of frequency. In the case of quasiperiodicity the frequency spectrum is discrete; it is composed of harmonic sequences based on (in general) a finite number N of incommensurate fundamental frequencies. For a good introduction to this topic see Ott (1993).

Locked injected oscillators have a periodic output. The oscillator may lock (see, eg Kurokawa 1973) to a rational number (p/q, p and q integers) times the injected frequency f and are therefore periodic with at most a period q/pf which may be rich in harmonics. Thus if we consider the nth harmonic of the locked oscillator, it lies at frequency npf/q and when mixed with the injected signal f in a nonlinear detector diode one expects to see a harmonic sequence of lines at integer multiples of (np/q- 1)f frequency. In addition the detector produces a harmonic sequence from the injection signal so in general we would not be surprised to see lines at frequencies (np/q-m)f with m,n,p and q all integers. The sum of the outputs from several free-running incommensurate oscillators is certainly a quasiperiodic signal with a discrete frequency spectrum. In the case of two incommensurate oscillators, part of the output from a detector diode driven by the composite signal will be periodic at the difference frequency. However, if there is harmonic content in these oscillators, there is no reason for the difference frequency to be commensurate with either of the original frequencies and so the result of applying the signal to a detector is to give an output having a discrete frequency spectrum with incommensurate frequency components. Thus the trajectory in phase space does not close.

If the injected oscillator is driven into a chaotic state it will have a continuous frequency spectrum which will be frequency shifted in the detector by an amount equal to the frequency of the injected signal. Observation of a continuous spectrum in the detector output (as is reported below) we hypothesize is an indication of chaos in the injected ocillator output. In this paper we address the issue; is there chaos in injected Gunn sources under certain conditions? If the motion is true chaos then we can make some observations about the possibility of certain properties of the frequency spectrum of the signal so produced, including its stationarity or non-stationarity. There may be intermittencies, where periodic solutions of irregular length follow bursts of chaos. There may be competing chaotic attractors between which the motion passes irregularly and intermittently; in this case the observed "spectrum" may change erratically with time, even in the absence of controlling parameter variations. Multiple solutions of a nonlinear system commonly coexist; thus the behaviour of the source in the long term depends on the route by which the controlling parameters achieve their steady state values; also the oscillator can be impulsively kicked from one stable chaotic solution to another, or even between periodic locked behaviour and chaos.

These considerations suggested an experiment to look for possible chaos in an injected Gunn oscillator. The experimental setup used is described here and shown in figure 3: an X band klystron is coupled into X band waveguide through a 27+/-5 dB isolator to prevent reverse interaction of the output of the Gunn source on the klystron. The waveguide feeds an attenuator and then an E plane Tee junction, on the two arms of which are mounted respectively a tuneable Gunn oscillator at about 10 GHz and a crystal detector. The detector output is monitored with a spectrum analyser having a minimum resolution bandwidth of 50 kHz.

Both the amplitude of the source oscillator and the frequency of the injected oscillator can be varied. For sufficiently small injection power the oscillators run freely; the ouput of the spectrum analyser consists of small amounts of intermodulation product at the difference frequency. The sum of the output of the klystron and the Gunn source is almost certainly quasiperiodic, in the sense that on continuous variables one is very unlikely to choose two points at random whose ratio is a rational number, i.e. may be expressed as the ratio of two integers.

As the amplitude is increased, the spectrum of figure 1 emerges at a well defined threshold; it appears that a bifurcation has happened. The figure shows a series of discrete lines at roughly 5 MHz spacing, with quite complicated structure in the detail. We hypothesize that the details of the structure may be explained by postulating that the mode lock is slipping between close integer ratios, during the sweep time of the spectrum analyser. Only a single sweep is displayed. The basic 5 MHz frequency spacing we interpret as mode locking at a ratio of frequencies of the order 2000:2001. It is also possible that the difference between the injected signal and the Gunn source is of the order of 5 MHz which is generating harmonics in the detector diode; however, the high initial frequencies (10GHz) and the relative instability of the Gunn source would mean that this difference frequency would vary continuously over time, whereas in fact there is evidence from close examination of pictures like figure 1 that it shifts in frequency erratically and discontinuously.

As the amplitude is varied further around this commensurate frequency lock the continuous spectrum of figure 2 emerges at a further bifurcation threshold. Examination of this spectrum at the minimum resolution bandwidth of the spectrum analyser of 50 kHz exhibits no trace of a discrete spectrum.

Any commensurate frequency lock would have to be at a closer ratio than 200,000 to 200,001. It is also possible that the commensurate lock is slipping at a rate fast enough to generate the observed trace on the spectrum analyser which is sweeping relatively slowly. However, this begs the question as to whether the mode slippage is regular and periodic, or erratic, and whether it is due to parameter variation (temperature, voltage, dimensions).

It is hard to be certain that the motion is truly chaotic from the frequency spectrum alone; ideally one would like to acquire a Poincare section of the motion and look for the fractional dimension of any attractor. At 10 GHz this is at present impracticable.

This observation of possible chaos in injected Gunn sources must be ubiquitous in systems wherein it is attempted to reduce the phase noise by injection locking a noisy high power device to a quiet low power device.

It is of course possible to produce simulations which predict chaotic states in Transferred Electron Devices from the analysis of model equations which underlie the behaviour.

references

Kurokawa,K
"Injection locking of microwave solid state sources"
Proceedings IEEE Oct 1973 vol 61 number 10 pp 1386-1410
Ott, E.

”Chaos in dynamical systems”
Cambridge University Press 1993

Figure captions

fig 1

The detector output spectrum from 0-100MHz at resolution bandwidth of 300kHz, showing evidence for commensurate frequency mode locking which is slipping dynamically within the sweep time of the spectrum analyser. Both injected and Gunn oscillator frequencies are near 10 GHz.

fig 2

The detector output spectrum for a slightly different injected power, showing no evidence for a discrete spectrum at a resolution bandwidth of 100kHz.

fig 3

The experimental arrangement.